7 Cost of Production

ECO202 Fall 2019

October 22, 2019

Outline

1. What are Costs?
2. Cost-minimizing Input Choice
3. Economies of Scope
4. Learning Curve

What are Costs?

Costs are the resources used to make Q

Cost: payment for all inputs (at opportunity cost)

Cost is expressed **per** something

General cost function: C=f(Q)

Distinguish:

- Fixed cost (does not change with Q)
- Variable cost (varies with Q)

In the short run, at least one factor is fixed

Cost budget version

$$C = (w \times L) + (r \times K)$$

w: wage paid per worker L

r: rental rate per unit of capital K

Marginal cost is key

Marginal Cost is the increase in cost from the production of one extra unit of output

$$MC = \Delta C / \Delta Q$$

$$C = 100 + 12Q^2 \implies MC = 24Q$$
$$C = 50Q + 3Q^3 \implies MC = 50 + 9Q^2$$

Describing marginal cost

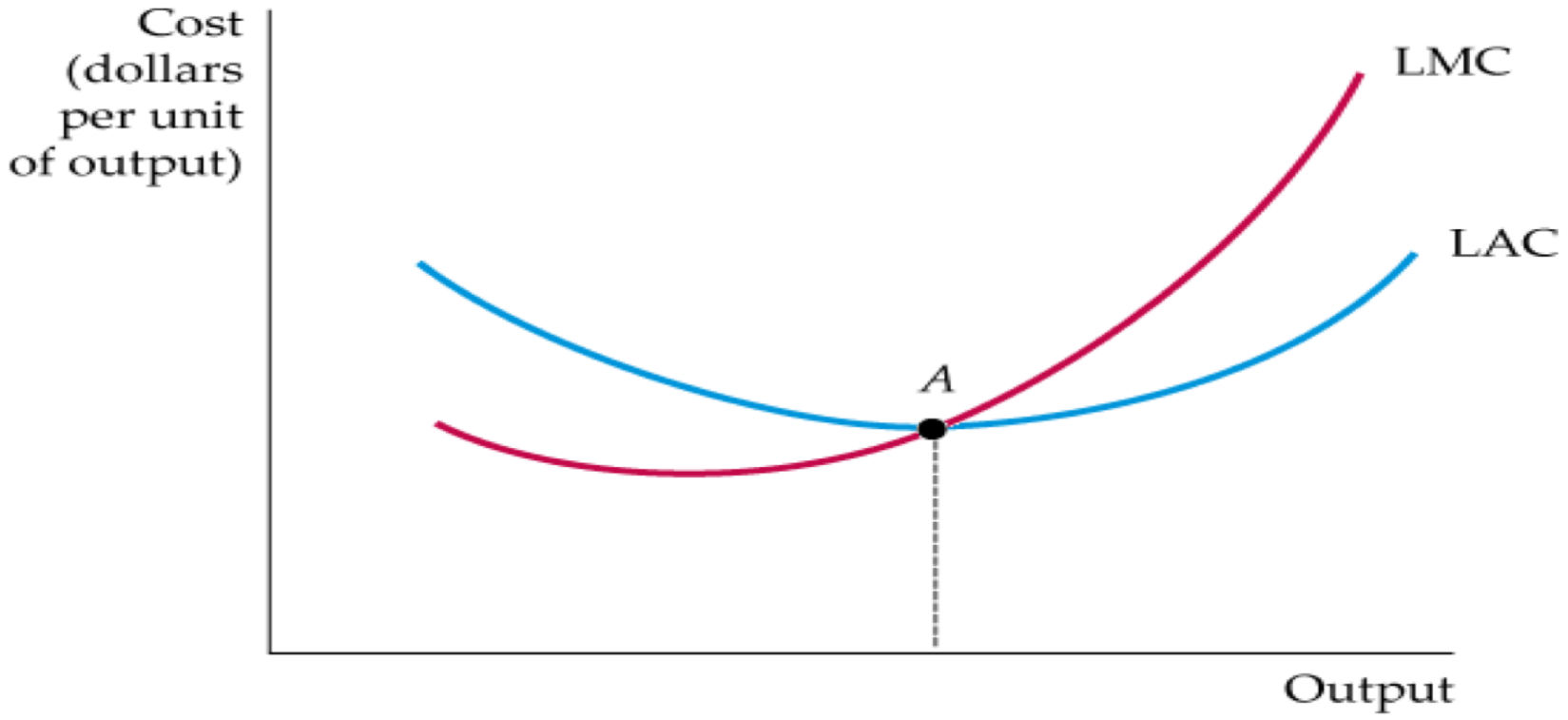
If K is fixed:

$$C = (w \times L) + (r \times K)$$
$$MC = w\Delta L/\Delta Q$$
$$MC = w/(\Delta Q/\Delta L)$$
$$MC = w/MP_L$$

MC depends on the rate of change of output with respect to labor multiplied by the wage

As MP_L is diminishing, MC is upward-sloping

Long run marginal and average cost



Cost-minimizing Input Choice

Cost minimization

Fundamental challenge all firms face:

How to **select input bundle** to produce a given output Q at minimum cost C

Same thing as:

How to maximize π

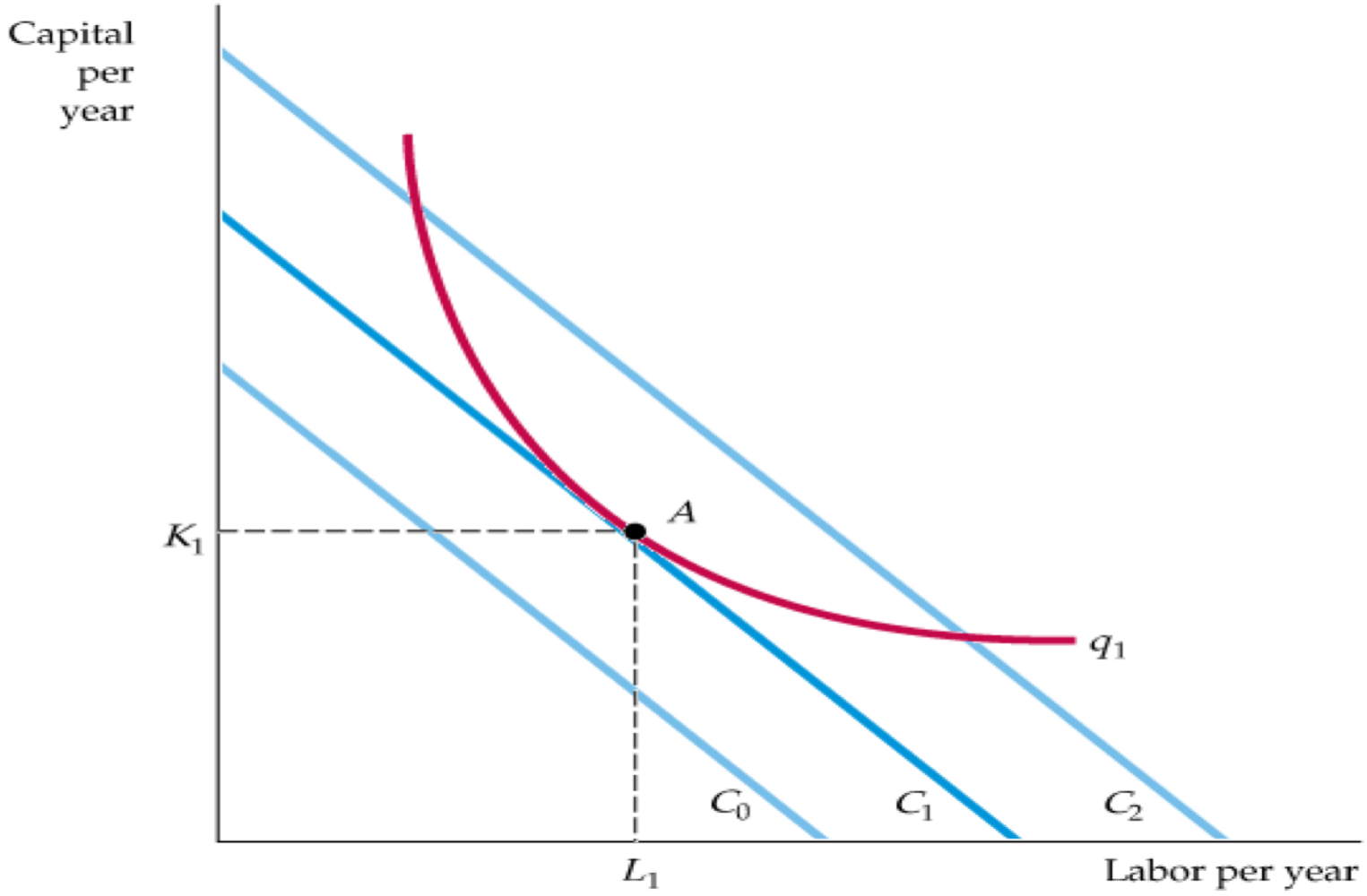
Constant-cost line

Isocost line: locus of all combinations of K and L that can be purchased for a given total cost $\bar{C}$

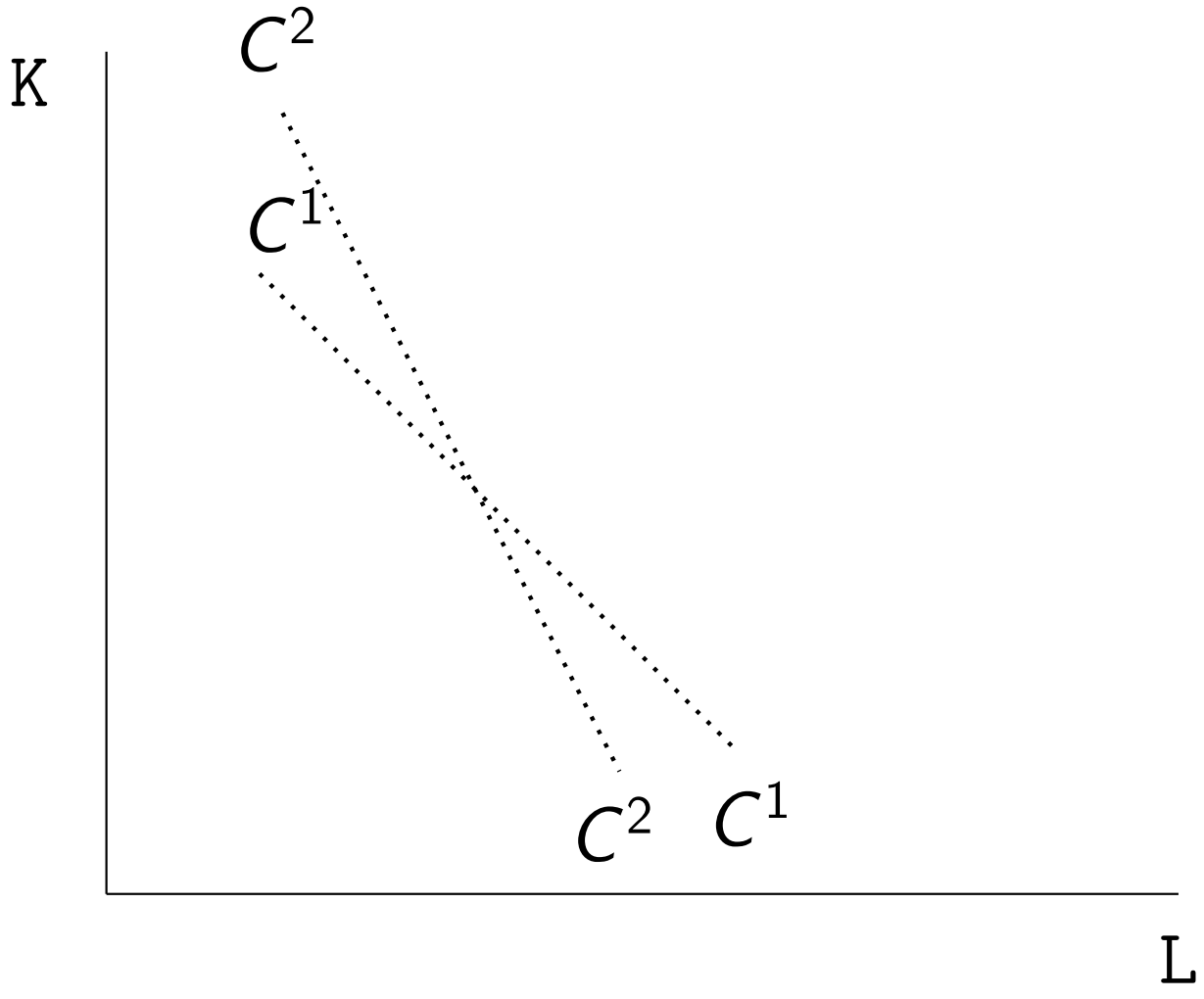
$$\bar{C} = wL + rK$$
$$K = \bar{C}/r - (w/r)L$$

Draw this in (L, K) space; it has slope –w/r

Isocost lines in blue



Relative input price change



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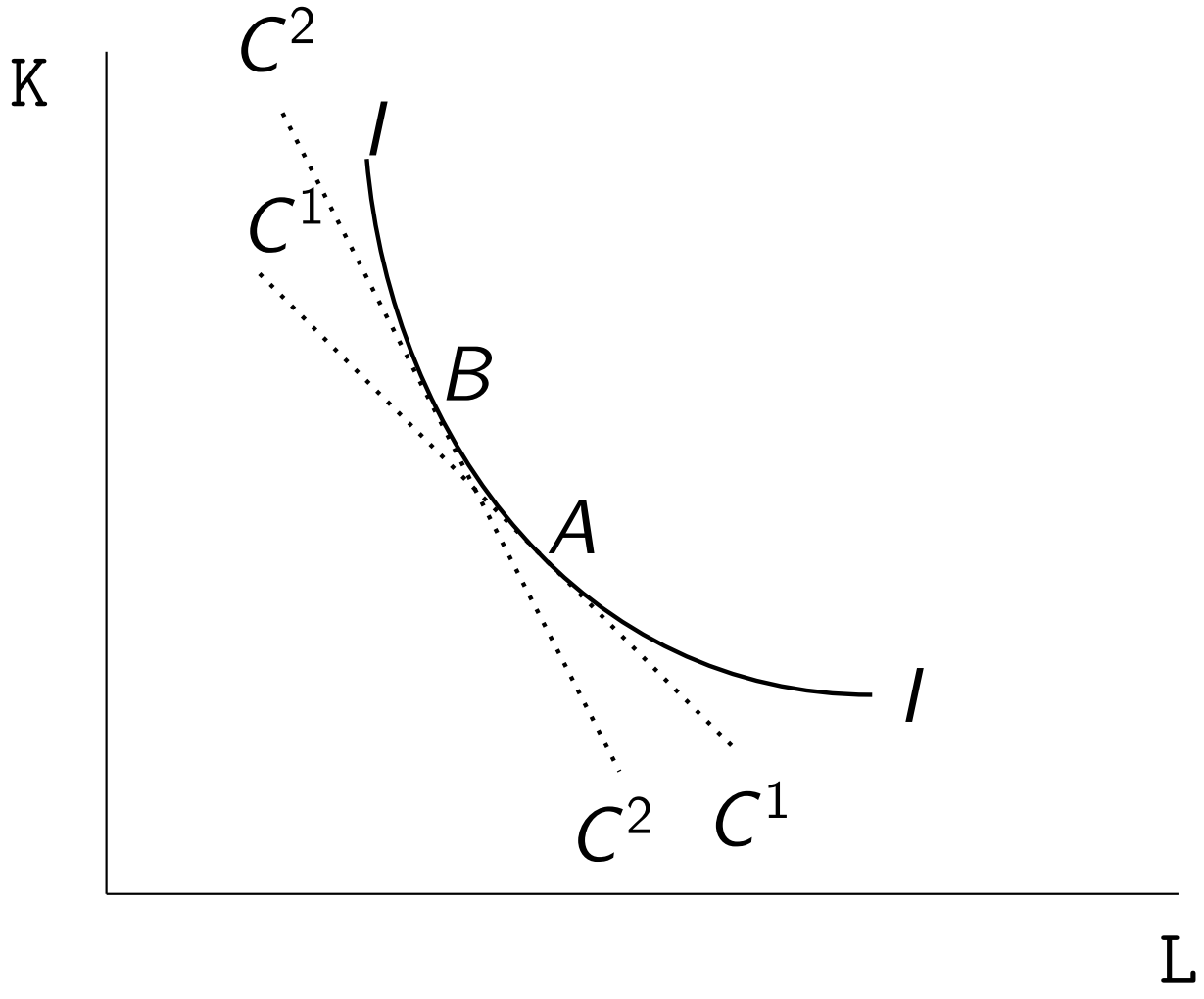

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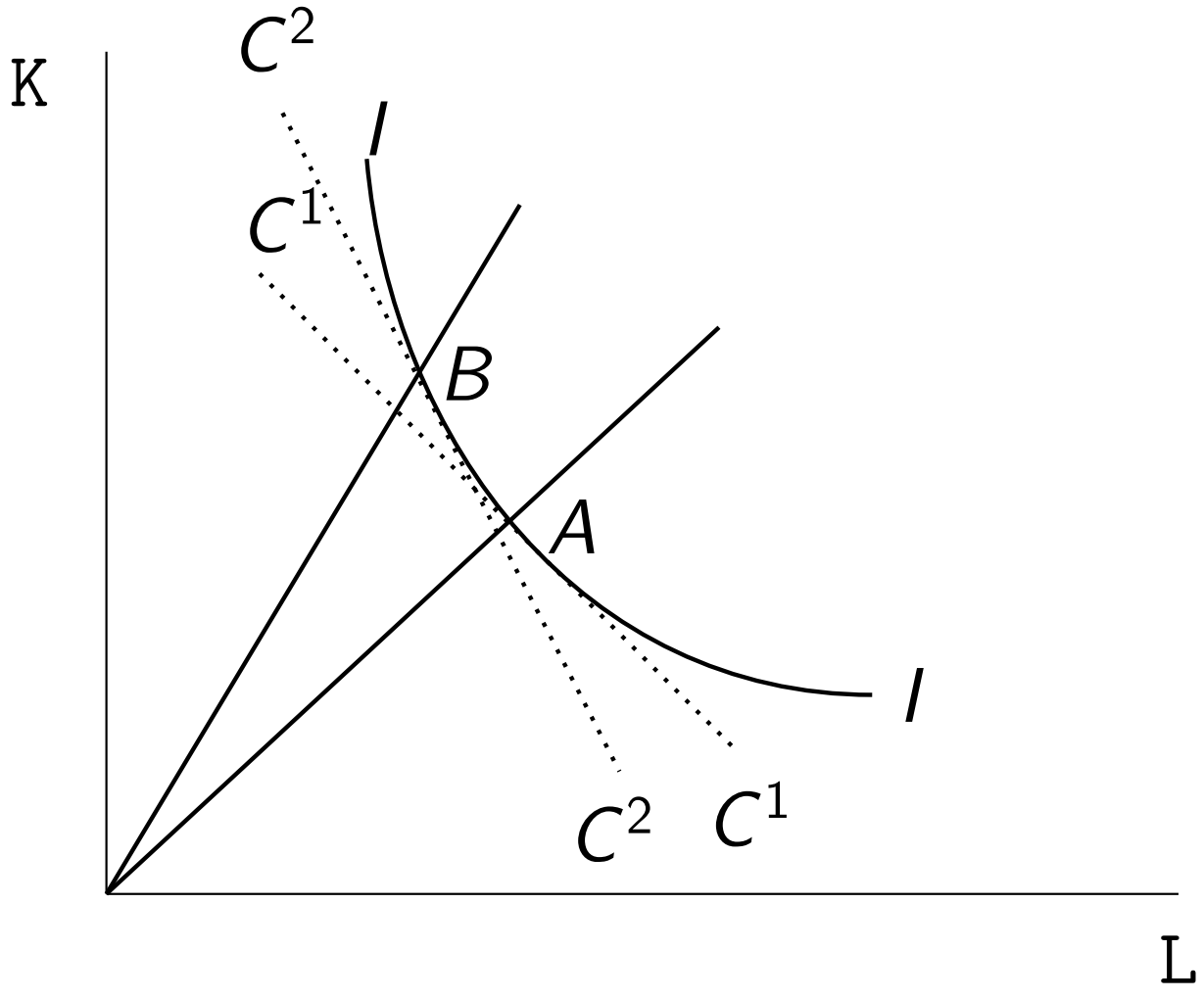


Cost-minimizing conditions

Isocost line must be tangent to the furthest out isoquant

Slope of isoquant equals slope of isocost line:

$$-MP_L/MP_K = -w/r$$
$$MP_L/MP_K = w/r$$
$$MP_L/w = MP_K/r$$

Intuitive: equalized productivity to input prices

Solving for optimal input choice

Max Q=f(K, L) when C=wL + rK

Two equations:

$$MP_L/MP_K = w/r \quad (1)$$

$$C = wL + rK \quad (2)$$

Solving for two unknowns (K^*, L^*)

Factor demand functions

Q=f(K,L)=6K$^2L^3$

Equation (1):

$$\frac{18K^2L^2}{12KL^3} = \frac{w}{r} \quad \rightarrow \frac{3K}{2L} = \frac{w}{r} \quad \rightarrow K = \frac{2Lw}{3r}$$

Put in the cost constraint:

C=wL + rK=wL + r(2Lw/3r)

Solve for L $\rightarrow L^* = 3C/5w$

Solve for K $\rightarrow K^* = 2C/5r$

Cost function

$Q=f(K,L)=6K^2L^3$

Put in factor demand functions:

$$Q=6(2C/5r)^2(3C/5w)^3$$

Just math

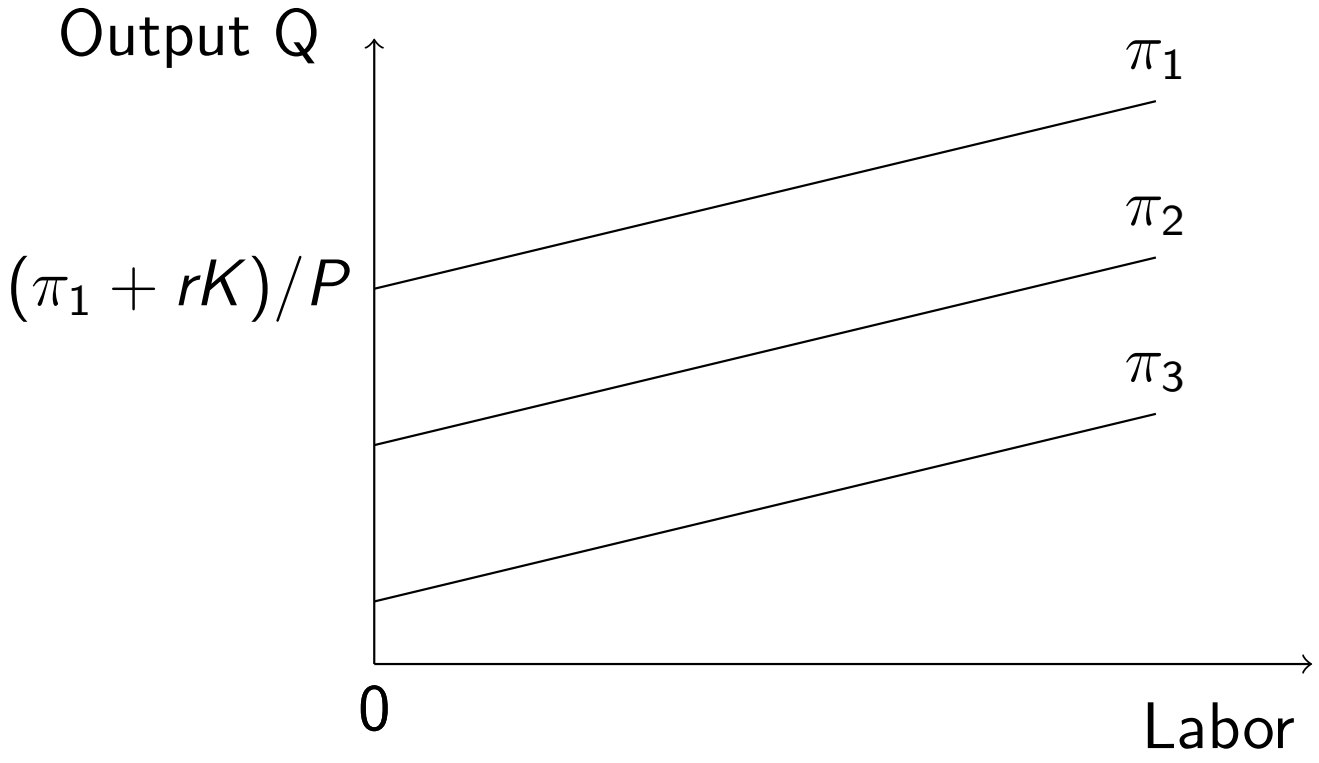
$$C = \left(\frac{Qw^3r^2}{0.207}\right)^{0.2}$$

Profit function

$$\pi = TR - TC$$
$$\pi = (P \times Q) - TC$$
$$\pi = (P \times Q) - (wL + rK)$$
$$PQ = \pi + (wL + rK)$$
$$Q = (\pi + rK)/P + (w/P)L$$

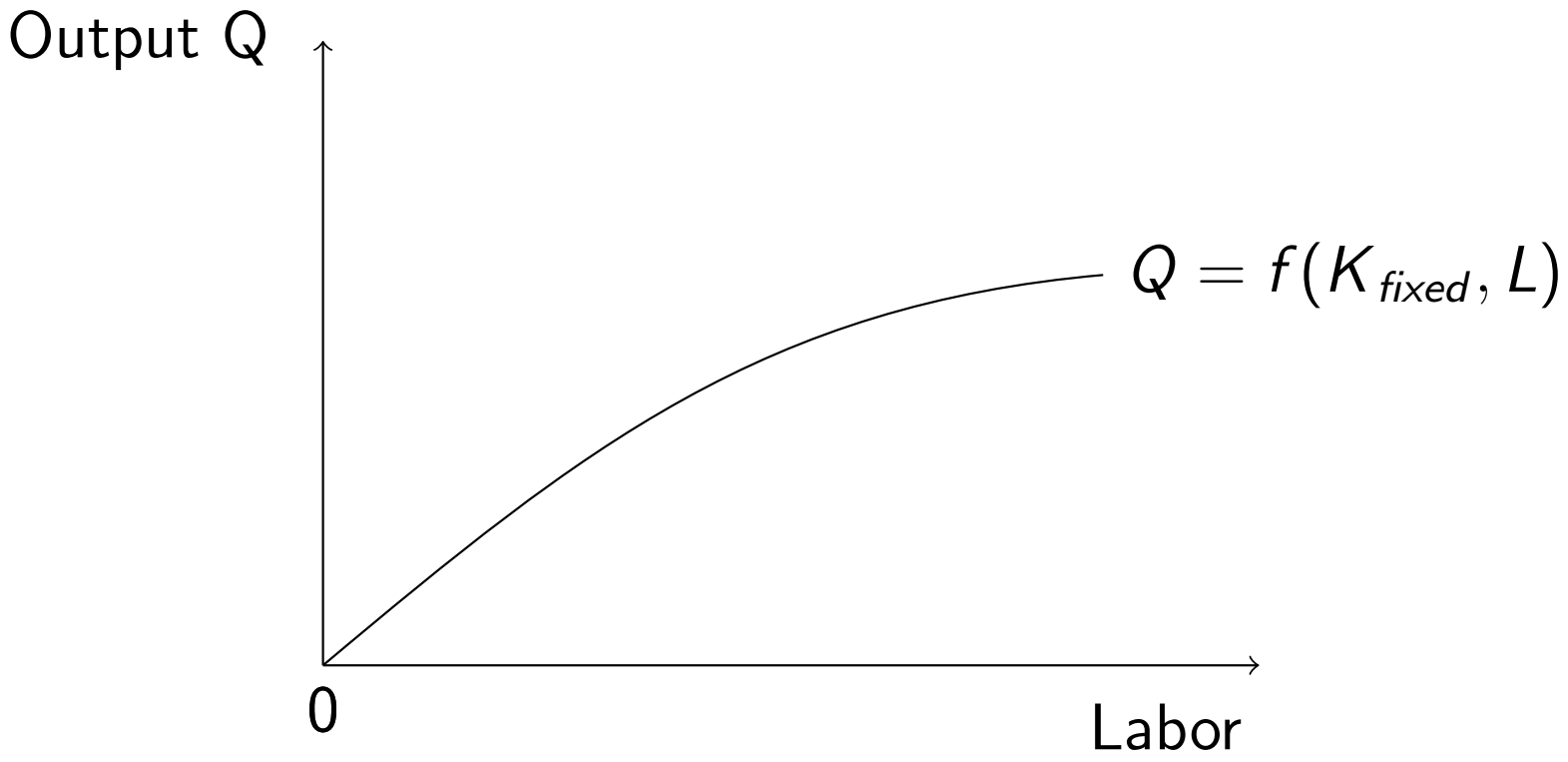
If K is fixed, plot in (L,Q) space for given π

Isoprofit lines



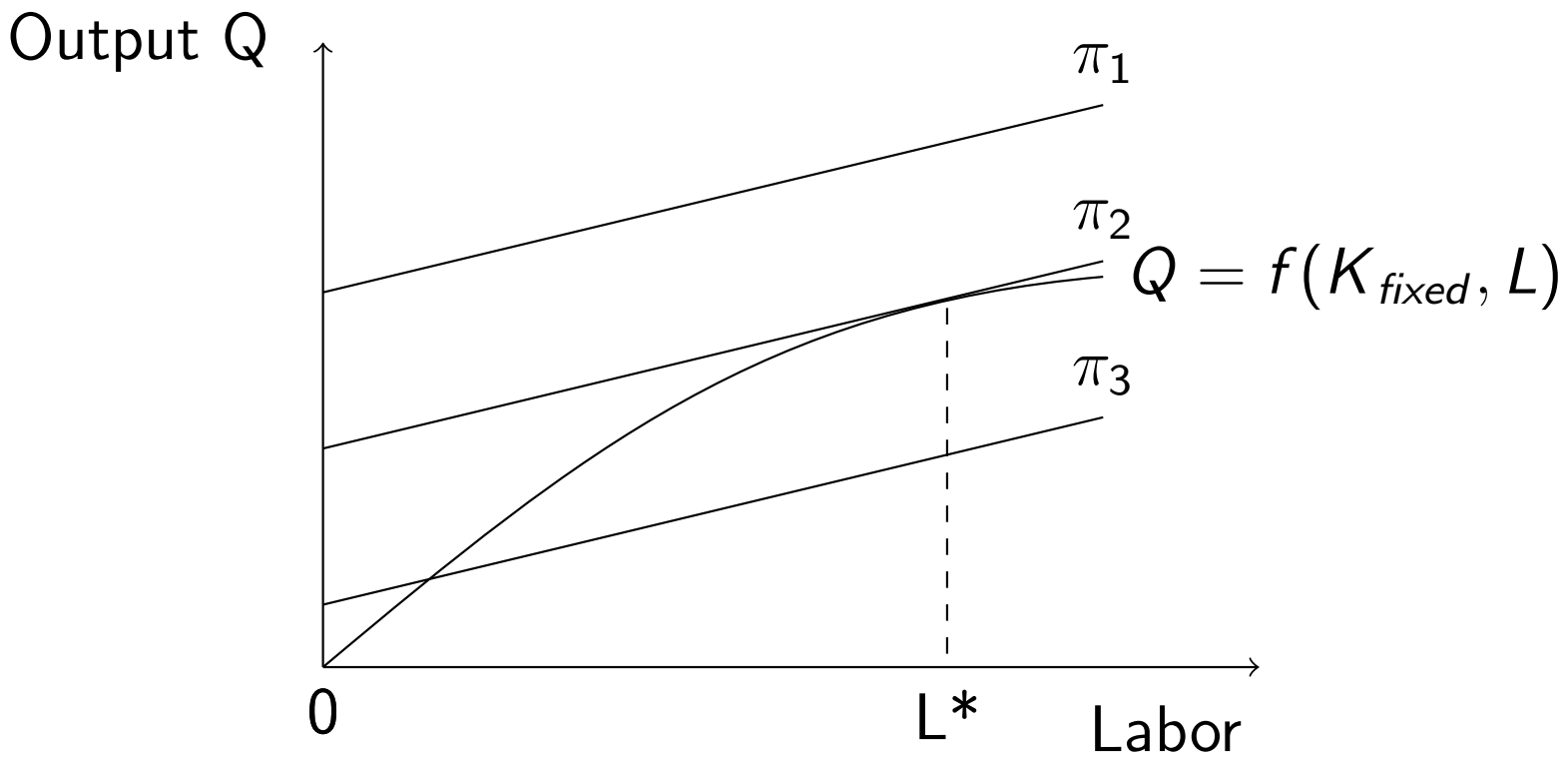
For given π, K, and P, isoprofit line slope is (w/P)

Production function



Slope of production function is MP_L

Optimality



Optimal labor input

Optimal labor input choice: when isoprofit line is tangent to the production function

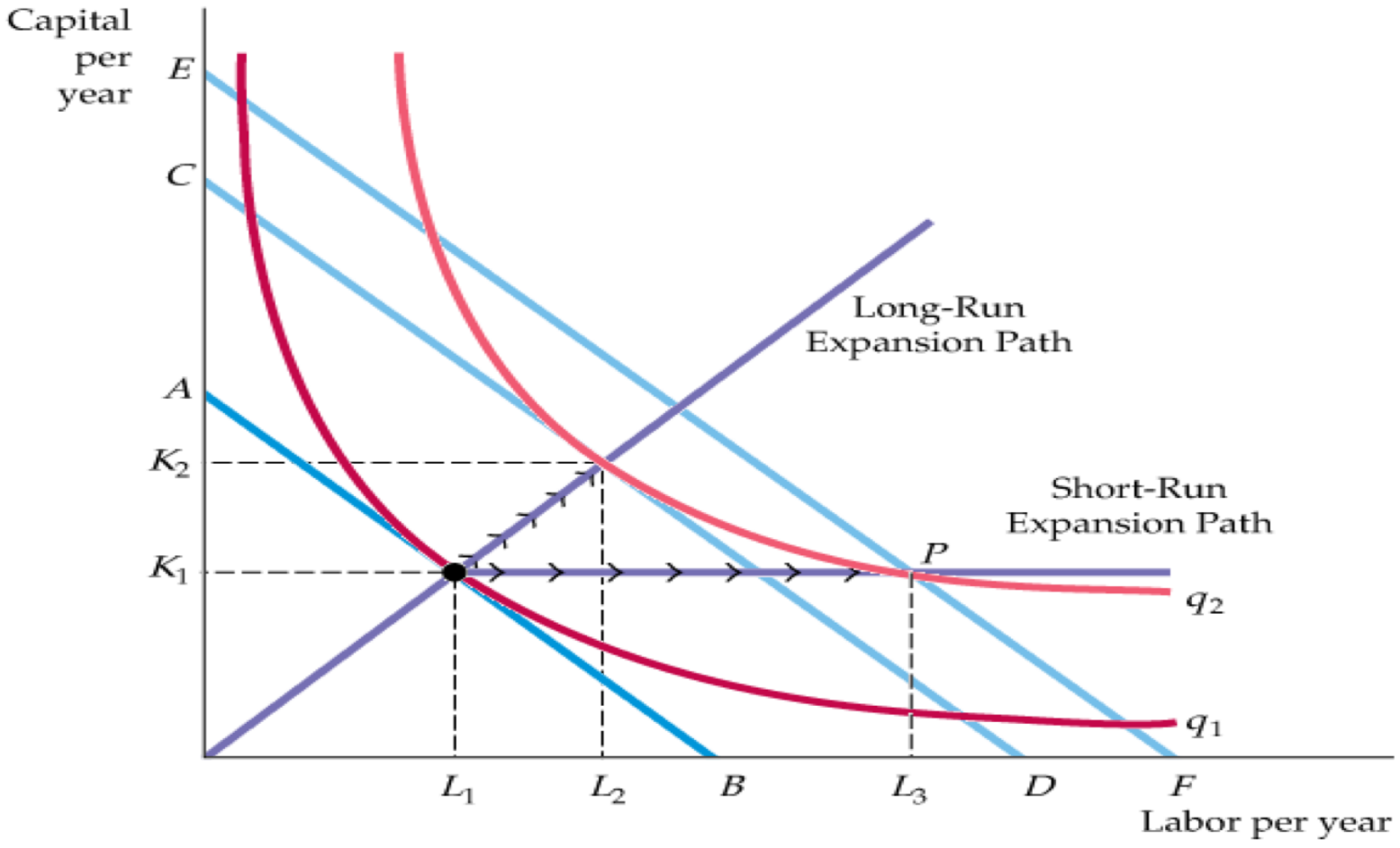
Slope of isoprofit line is w/P

Slope of production function is MP_L

Equilibrium:

$$w/P = MP_L$$

Short and long-run cost adjustments



Economies of Scope

Why only make one product?

Firms may make more π if they make multiple products:

- Learning from production of first product
- Diversification (umbrellas and parasols)
- Bundling or tying
- Upselling

How to decide how many products to make?

Are there **economies of scope**?

Economies of scope happen when joint output is greater in a single firm that two separate firms (with same inputs in total)

Degree of economies of scope:

$$\Phi = [C(Q_1) + C(Q_2) - C(Q_1, Q_2)]/C(Q_1, Q_2)$$

Example

Tariff at dentist:

X-ray=\$50 Clean=\$70 Combo=\$100

$\Phi = (70 + 50 - 100)/100 = 0.2$

Good news: Stark Law limits upselling

Learning Curve

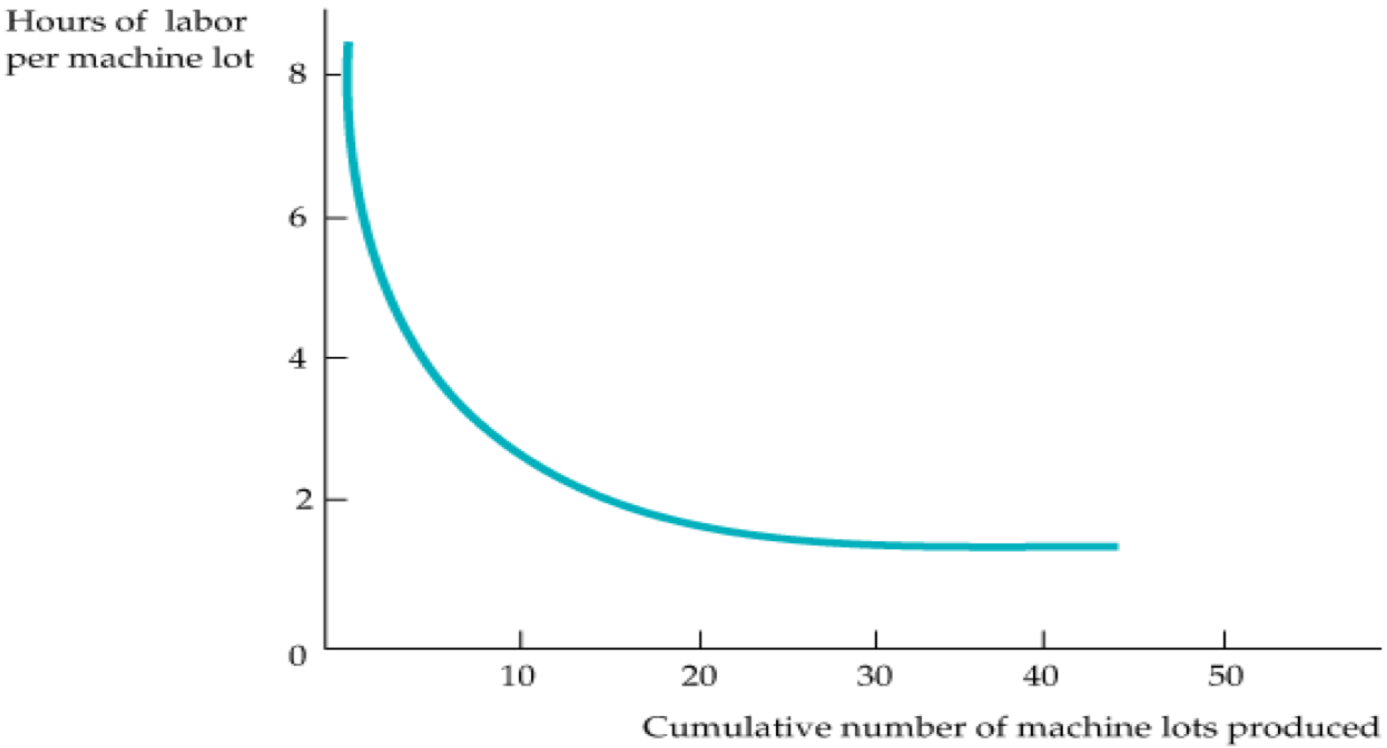
Learning makes for efficiency

Over time, **humans learn**:

- Workers take longer on tasks the first few times; as they practice, they get faster
- Managers learn to schedule the production process better (e.g., just-in-time inventory)
- Engineers are initially cautious in product design; with experience they allow for lower tolerances and develop more specialized tools

With learning, the firm's MC and AC fall

Learning is cumulative



Learning curve equation

Formulate the learning curve as:

$$L = A + BN^{-\beta}$$

L: labor input

N: cumulative output over time

A, B, and β are parameters; A>0, B>0, $\beta \in [0, 1]$

Predicting learning

$$L = A + BN^{-\beta}$$

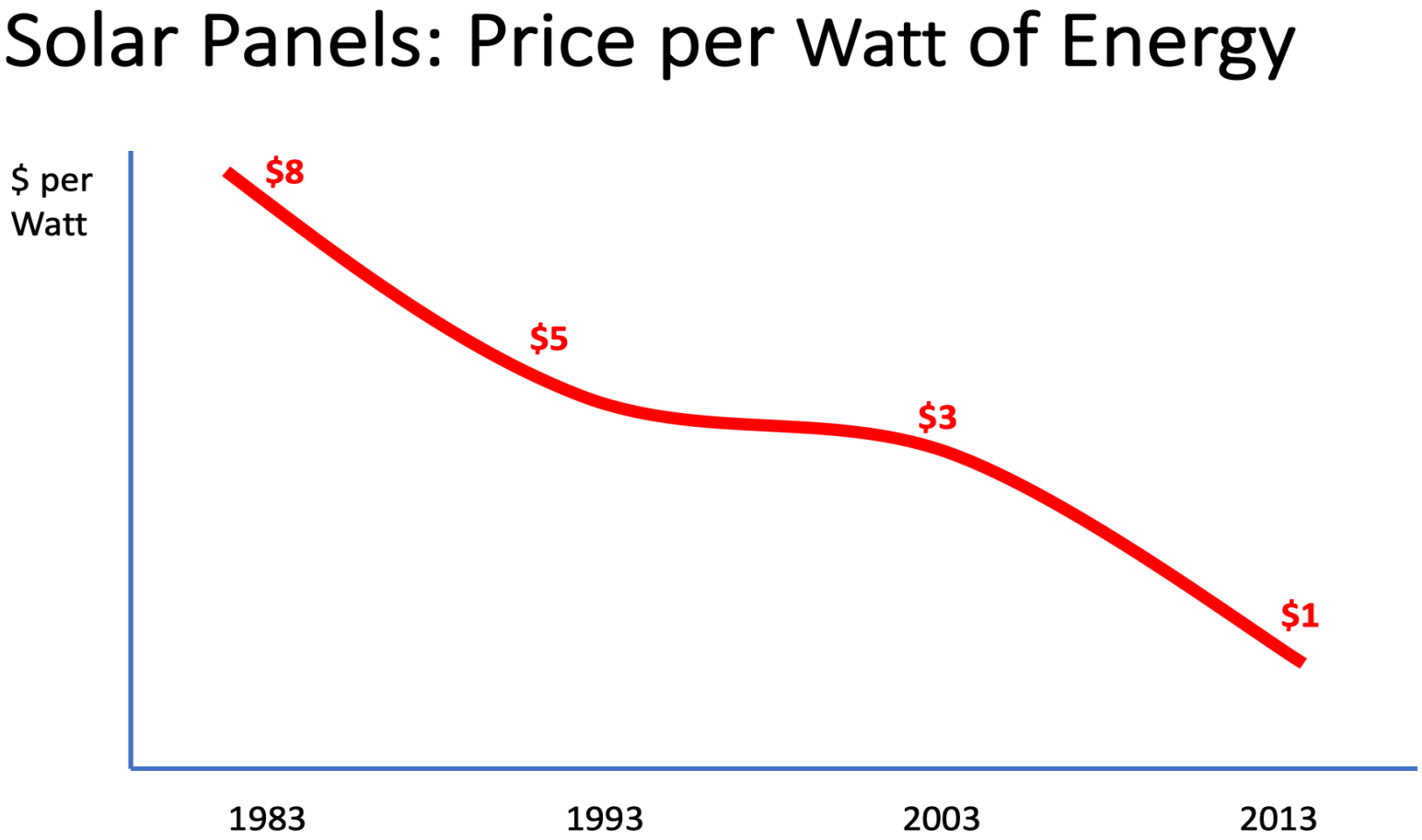
If first unit made, N=1: $\rightarrow L = A + B$

If no learning $\beta = 0$, for all N: $\rightarrow L = A + B$

If learning $\beta > 0$, as N$\uparrow$: $\rightarrow L \approx A$

As $\beta \uparrow$, learning curve shifts down faster

Example: Solar power



Example: Electric cars

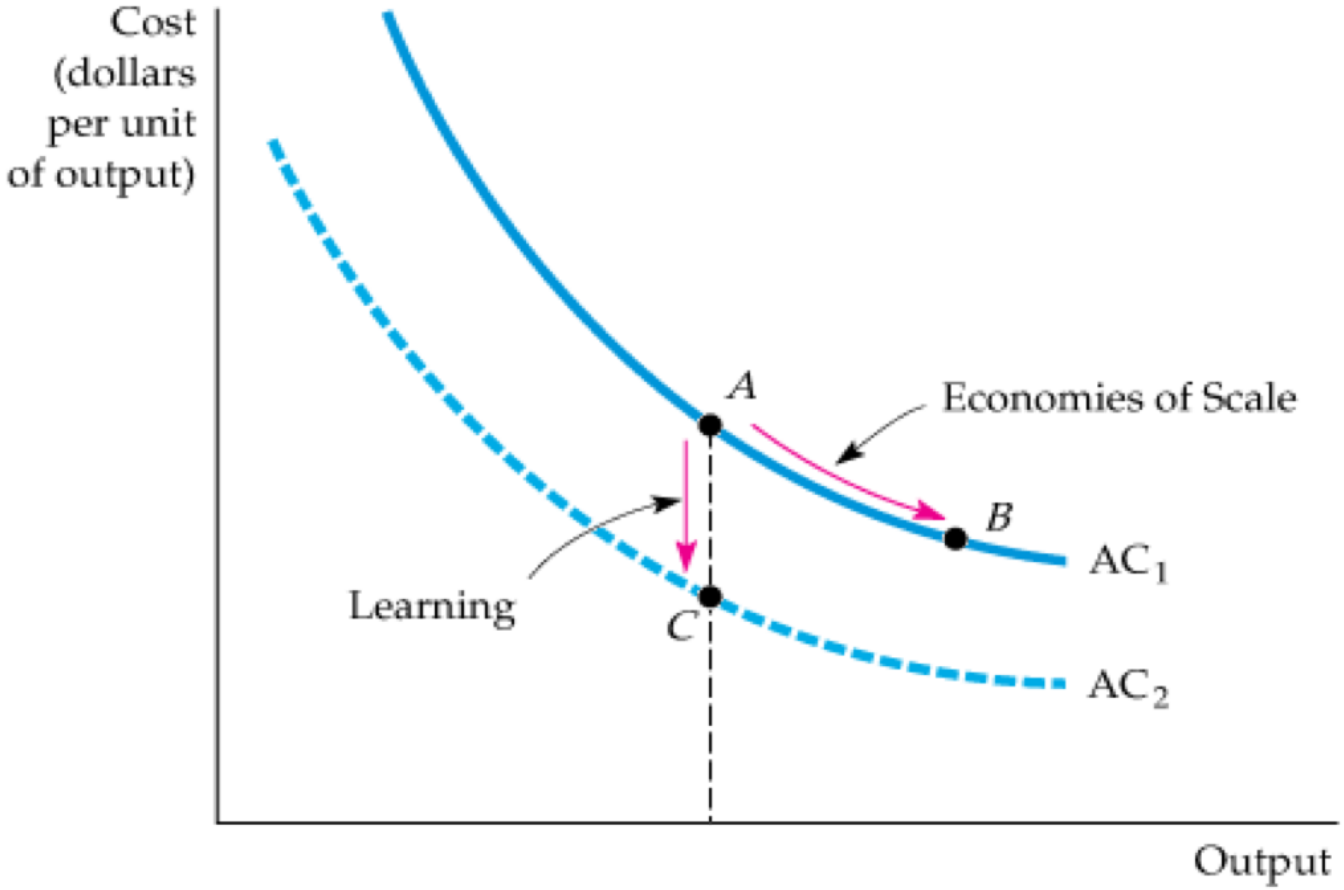
Electric Cars go about 250 miles per charge

Tesla Roadster 2011 **$110,000**

Tesla Model 3 2013 **$80,000**

Tesla Model X 2017 **$45,000**

Learning is different from economies of scale



Any Questions?

What are Costs?

Cost-minimizing Input Choice

Economies of Scope

Learning Curve